UG–447	**BMS–21**

B.Sc. DEGREE EXAMINATION –
DECEMBER, 2018.

Second Year

Mathematics

GROUPS AND RINGS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Define residue classes of integers mod n.
2. Let G be a group. Then Prove that (a) identity clement is unique (b) inverse element of every element is unique.
3. Define left coset and right coset.
4. Define the isomorphism between two rings.
5. Define left ideal and right ideal of the ring R.
6. Prove that a finite integral domain is a field.
7. Prove that Every field F is an integral domain.
8. Prove that Every proper ideal I is the kernel of a ring homomorphism.

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. (a) If H and K are subgroups of a group G then show that $H \cap K$ is also subgroup of G.

   (b) Prove that a subgroup of a cyclic group is cyclic

10. Let G be a group and H is the subgroup of G, then show that

    (a) $aH = H \Leftrightarrow a \in H$

    (b) $aH = bH \Rightarrow a^{-1}b \in H$

    (c) $aH = bH \Leftrightarrow a \in bH$

    (d) $a \in bH \Rightarrow a^{-1} \in Hb^{-1}$

    (e) $aH = Ha \Leftrightarrow H = aHa^{-1}$

11. State and prove Lagrange's theorem on finite group.

12. State and prove the fundamental theorem of homomorphism.

13. Prove that any integral domain can be embedded in a field.

14. Let R be a ring and $a, b \in R$, then show that

(a) $0a = a0 = 0$

(b) $a(-b) = (-a)b = -(ab)$

(c) $(-a)(-b) = ab$

(d) $a(b-c) = ab - ac$

15. (a) Show that a ring R has no zero devisor iff cancellation law is valid in R.

(b) Show that any unit in R cannot be a zero devisor.

16. Derive the necessary and sufficient condition for a subset H of a group G to be a subgroup of G.

———————