UG–656 **BMS 32/BMC 32**

B.Sc. DEGREE EXAMINATION —
JUNE 2018.

Third Year

Mathematics

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Let R^+ be the set of all positive real numbers. Define addition and scalar multiplication as follows:

   (a) $u + v = uv$ for all $u, v \in R^+$

   (b) $au = u^{\alpha}$ for all $u \in R^+$ and $\alpha \in R$.

   Prove that R^+ is a real vector space.

2. Show that the mapping $T : V_2(R) \to V_3(R)$ defined by $T(a, b) = (a + b, a - b, b)$ is a linear transformation.

3. Let V be a vector space over a field F. Show that any subsets of V containing the zero vector is linearly independent.

4. Show that every linearly independent subset of a finite dimensional vector space V forms a part of a basis.

5. Let $S = \{v_1, v_2, ...v_n\}$ be an orthogonal set of non — zero vectors in an inner product space V. Show that S is linearly independent.

6. Let f be the bilinear form defined on $V_2(R)$ by $f(x, y) = x_1y_1 + x_2y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of f with respect to the standard basis $\{e_1, e_2\}$.

7. Define partial order relation on a set and give an example. Further check whether the relation 'a divides b' is a partial order on the set Z of integers.

8. Prove that any distributive lattice L is a modular lattice.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. State and prove fundamental theorem of vector space homomorphism.

10. Let V be a vector space over a field F. Let $S, T \subseteq V$. Prove that

(a) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

(b) $L(SUT) = L(S) + L(T)$

11. Let V and W be vector spaces over a field F. Let $T : V \rightarrow W$ be an isomorphism. Prove that T maps a basis of V onto a basis of W.

12. Let V and W be two finite dimensional vector space over a field F. Let $\dim V = m$ and $\dim W = n$. Show that $L(V, W)$ is a vector space of dimension mn over F.

13. Apply Gram — Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$. where $v_1 = \{1, 0, 1\}$. $v_2 = \{1, 3, 1\}$ and $v_3 = \{3, 2, 1\}$.

14. Show that the set of all bilinear forms on a vector space V is also a vector space over F.

15. Reduce the quadratic form $x_1^2 + 4x_1x_2 + 4x_1x_3 + 4x_2^2 + 16x_2x_3 + 4x_3^2$ to the diagonal form.

16. (a) Let B be a Boolean algebra. Show that $(a \vee b)' = a' \wedge b'$, $(a \wedge b)' = a' \vee b'$ and $(a')' = a$.

(b) In a Boolean algebra if a $a \vee x = b \vee x$ and $a \vee x' = b \vee x'$ Show that $a = b$.

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