UG–646 **BMS–11/ BMC–11**

M.A. DEGREE EXAMINATION – JUNE, 2018.

First Year

Mathematics

ELEMENTS OF CALCULUS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Find the n^{th} differential co-efficient of $\cos x \cos 2x \cos 3x$.

2. Find the maximum value of function $f(x, y) = xy(a - x - y)$.

3. Find the radius of curvature at the point 't' of the curve $x = a(\cos t + t \sin t)$ $y = a(\sin t - t \cos t)$.

4. Evaluate $\int_0^{\pi/2} \sin^7 \theta \cos^5 \theta \, d\theta$.

5. Find the area of the cardiod $r = a(1 + \cos\theta)$.

6. If $\{s_n\}$ is a sequence of non-negative numbers and if $\lim_{n\to\infty} s_n = L$, then $L \geq 0$.

7. Prove $\lim_{n\to\infty} \frac{3n^2 - 6n}{5n^2 + 4} = \frac{3}{5}$.

8. Show that the series $\sum_{n=1}^{\infty} 2n/(n^2 - 4n + 7)$ diverges.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If $y = \left(x + \sqrt{1 + x^2}\right)^m$, prove that $(1 + x^2) y_{n+2} + (2n + 1) x y_{n+1} + (n^2 - m^2) y_n = 0$.

10. (a) If $u = \frac{xy}{x + y}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u$.

(b) $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.

11. Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

12. Prove that the radius of curvature at a point $(a\cos^3\theta, a\sin^3\theta)$ on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $3a\sin\theta\cos\theta$.

13. Find the length of one loop of the curve $3ay^2 = x(x-a)^2$.

14. Establish the reduction formula for $\int \sin^n x\,dx$ and hence evaluate $\int_0^{\pi/2} \sin^6 x\,dx$.

15. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.

16. If $\{s_n\}$ is a sequence of real number which converges to L then show that $\left\{s_n^2\right\}_{n=1}^{\infty}$ converges to L^2.
