PG–387	**MMS–15/ PGDMAT–11**

M.Sc. DEGREE/P.G. DIPLOMA EXAMINATION – JUNE, 2018.

First Year

Mathematics

ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let G be a group in which $(ab)^m = a^m b^m$ for three consecutive integers and for all $a, b \in G$. Prove that G is abelian.

2. Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

3. Prove that a finite integral domain is a field.

4. If U, V are ideals of R, let $U+V = \{u+v : u \in U, v \in V\}$. Prove that $U+V$ is also an ideal of R.

5. If V is a finite-dimensional space over F, prove that any two bases of V, have the same number of elements.

6. If V is a vector space and $u,v \in V$, then prove that $|(u,v)| \le \|u\|\|v\|$.

7. If L is a algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

8. If V is finite-dimensional over F, prove that $T \in A(V)$ is regular if and only if T maps V onto V.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. State and prove first part of Sylow's theorem.

10. State and prove Cayley's theorem.

11. If R is a ring with unit element, then for all $a,b \in R$ prove that

(a) $a.0 = 0.a = 0$

(b) $a(-b) = (-a)b = -(ab)$

(c) $(-a)(-b) = ab$

(d) $(-1)a = -a$

(e) $(-1)(-1) = 1$.

12. Prove that every integral domain can be imbedded is a field.

13. If $v_1, v_2, ..., v_n$ is a basis of V over F and if $w_1, w_2, ... w_m$ in V are linearly independent over F, prove that $m \leq n$.

14. If V and W are of dimensions m and n respectively over F, then prove that $Hom(V, W)$ is of dimensions mn over F.

15. If F is of characteristic 0 and if a, b are algebraic over F, then prove that there exist an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.

16. If $T \in A(V)$ has all its characteristic roots is F, then prove that there is a basis of V is which the matrix of T is triangular.
