PG–390 **MMS–18**

M.Sc. DEGREE EXAMINATION — JUNE, 2018.

First Year

Mathematics

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let X have the probability density function $f(x) = \begin{cases} \dfrac{x+1}{2}, & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$. Find the mean and variance.

2. Let X have the probability density function $f(x) = \begin{cases} \dfrac{x^2}{9}, & 0 < x < 3 \\ 0, & \text{elsewhere} \end{cases}$. Find the probability density function of $Y = X^3$.

3. Let X equal the length of life of 60 watt light bulb marketed by a certain manufacturer of light bulbs. Assume that the distribution of X is $N(\mu, 1296)$. If a random sample of $n = 27$ bulbs were tested until they burned out, yielding a sample mean of $\overline{X} = 1478$ hrs. Find the 95% confidence interval for μ.

4. Let X be a random variable having the probability density function $f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$. Test the simple hypothesis $H_0 : \theta = 2$ against the alternative simple hypothesis $H_1 : \theta = 4$, use a random sample X_1, X_2 of size $n = 2$ and define the critical region $C = \{(x_1, x_2) / 9.5 \leq x_1 + x_2 < \infty\}$. Also find the power of the test.

5. Let $X_1, X_2, ... X_n$ denote a random sample from a poisson distribution, that has the mean $\theta > 0$. Prove that $\overline{X}$ is an efficient estimator of θ.

6. Find $P_r\left(0 < X_1 < \frac{1}{3}, 0 < X_2 < \frac{1}{3}\right)$, if the random variables X_1 and X_2 have the joint probability density function

$$f(x_1, x_2) = \begin{cases} 4x_1(1 - x_2), & 0 < x_1 < 1, \\ & 0 < x_2 < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

7. Let $\overline{X}$ denote the mean of a random sample of size 128 from a gamma distribution with $\alpha = 2$ and $\beta = 4$. Find $P_r(7 < \overline{X} < 9)$.

8. Let $X_1, X_2, ... X_n$ denote the observations of a random sample of size $n > 1$ from a distribution that is $b(1, \theta), 0 < \theta < 1$. Let $Y = \sum_{i=1}^{n} X_i$. Find the unbiased and minimum variance estimator of Y / n.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Find the moment generating function of Gamma distribution and also find its mean and variance.

10. Let X_1, X_2 be a random sample from the normal distribution $n(0,1)$. Show that the marginal probability density function of $Y = \dfrac{X_1}{X_2}$ is the Cauchy probability density function.

11. Let two random samples, each of size 10, from two independent normal distributions $n(\mu_1, \sigma_1^2)$ and $n(\mu_2, \sigma_2^2)$ yield $\overline{x} = 4.8, s_1^2 = 8.64$, $\overline{y} = 5.6$, $s_2^2 = 7.88$. Find a 95% confidence interval for $\mu_1 - \mu_2$.

12. State and prove Rao-Blackwell theorem.

13. Let X has a probability density function $f(x,\theta) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0,1 \\ 0, & \text{elsewhere} \end{cases}$. Using sequentially probability ratio test, test the hypothesis $H_0 : \theta = 1/3$ and $H_1 : \theta = 2/3$.

14. Let X and Y have bivariate normal distribution with parameters $\mu_1 = 3$, $\mu_2 = 1, \sigma_1^2 = 16, \sigma_2^2 = 25$ and $p = 3/5$. Determine the following.

    (a) $P_r(3 < Y < 8)$

    (b) $P_r(3 < Y < 8 / X = 7)$

    (c) $P_r(-3 < X < 3)$

    (d) $P_r(-3 < X < 3 / Y = -4)$.

15. Let Z_n be $\psi^2(n)$. Find the limiting distribution of the random variable $Y_n = \dfrac{Z_n - n}{\sqrt{2n}}$.

16. Let $X_1, X_2, X_3,,, X_n$ be a random sample from the exponential distribution with probability density function $f(x,\theta) = \frac{1}{\theta} e^{-x/\theta}, 0 < x < \infty$, $\theta \in \Omega = \{\theta / 0 < \theta < \infty\}$. Find the maximum likelihood estimator for θ.

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