PG–375	MMS–16

M.Sc. DEGREE EXAMINATION –
DECEMBER, 2018.

First Year

Mathematics

REAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. State and prove Schwarz inequality.

2. Prove that the set E is open if and only if its compliment is closed.

3. Prove that a mapping f of a metric space X in to a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

4. Let f be defined on $[a, b]$; if f has a local maximum at a point $x \in (a, b)$ and if $f'(x)$ exists then prove that $f'(x) = 0$.

5. If P* is a refinement of P, then prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$.

6. State and prove Cauchy criterion for uniform convergence.

7. Establish the relation between Gamma function and beta function.

8. Prove that a linear operator A on a finite dimensional vector space X is one-to-one if and only if the range of A is all of X.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that every k-cell is compact.

10. State and prove root test.

11. Let f be a continuous mapping of a compact metric space X in to a metric space Y, then prove that f is uniformly continuous on X.

12. State and prove Taylor's theorem.

13. Prove that $f \in R(\alpha)$ on $[a,b]$ if and only if for every $\varepsilon > 0$ there exist a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.

14. Prove that there exists a real continuous function on the real line is nowhere differentiable.

15. Derive Stirling's formula.

16. State and prove inverse function theorem.

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