PG–394	**MMS-28**

M.Sc. DEGREE EXAMINATION — JUNE, 2018.

Second Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let a_1, a_2 be constants and consider the differential equation $L(y) = y'' + a_1 y' + a_2 y = 0$. If r_1, r_2 are the distinct roots of the characteristic polynomial p where $p(r) = r^2 + a_1 r + a_2$ then prove that the function ϕ_1, ϕ_2 defined by $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = e^{r_2 x}$ are the solutions of the differential equation $L(y) = 0$.

2. Define Wronskian. Verify whether the functions $\phi_1(x) = e^{x}$ and $\phi_2(x) = e^{-x}$ are independent or not.

3. Prove that for any 'n', the coefficient of x^n in $P_n(x)$ is $\dfrac{(2n)!}{2^n (n!)^2}$.

4. Let $f(x)$ be periodic with period w. Let A be an $n \times n$ constant matrix. Then prove that a solution of $y' = Ay + f(x)$ is periodic of period w if and only if $y(0) = y(w)$.

5. Let f be continuous and satisfy a Lipschitz condition on a rectangle $R : |x - x_0| \leq a$, $|y - y_0| \leq b$ $(a, b > 0)$. If ϕ and φ are solutions of the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ on an interval I containing x_0, then prove that $\phi(x) = \varphi(x)$ for all $x \in I$.

6. Solve $\dfrac{\partial^4 z}{\partial x^4} + \dfrac{\partial^4 z}{\partial y^4} = \dfrac{\partial^4 z}{\partial x^2 \partial y^2}$.

7. Reduce the equation $\dfrac{\partial^2 z}{\partial x^2} + 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = 0$ into its canonical form.

8. Define :

   (a) Boundary value problem.

   (b) Interior and exterior Dirichlet problem.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. Let ϕ be any solution of $L(y) = y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \cdots + a_{n-1} y' + a_n y = 0$ on an interval I containing a point x_0. Then prove that $\| \phi(x_0) \| e^{-k|x-x_0|} \leq \| \phi(x) \| \leq \| \phi(x_0) \| e^{k|x-x_0|}$ for all x in I where $\| \phi(x) \| = \left(| \phi(x) |^2 + | \phi'(x) |^2 + \cdots + | \phi^{(n-1)}(x) |^2 \right)^{1/2}$ and $k = | a_1 | + | a_2 | + \cdots + | a_n |$.

10. Find the solution of the Initial value problem $y'' - 2y' + y = 2x$, $y(0) = 6$, $y'(0) = 2$.

11. Solve the Legendre equation using power-series technique.

12. Derive Bessel function of order 'n' of second kind.

13. Let $A(x)$ be an $n \times n$ matrix which is continuous on a closed and bounded interval. Then prove that there exists a solution to the initial value problem $y' = A(x) \cdot y$, $y(x_0) = x_0$, $(x_1, x_0 \in I)$ on I.

14. (a) Find a solution of $(D^2 - D')z = 2y - x^2$. (7)

    (b) Classify the equation $u_{xx} + u_{yy} = u_{xy}$. (3)

15. Discuss the method of solving hyperbolic equations.

16. State and prove Kelvin's Inversion theorem.

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