

**UG-328**

**BMS-11/  
BMC - 11**

**B.Sc. DEGREE EXAMINATION —  
DECEMBER, 2019.**

**First Year**

**Mathematics**

**ELEMENTS OF CALCULUS**

Time : 3 hours

Maximum marks : 75

**PART A — (5 × 5 = 25 marks)**

Answer any FIVE questions.

1. Find the  $n^{th}$  differential co-efficient of  $y = \sin^3 x$ .
2. Find the radius of curvature for the curve  $\sqrt{x} + \sqrt{y} = 1$  at  $(\frac{1}{4}, \frac{1}{4})$ .
3. Evaluate :  $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 s} dx$ .
4. Evaluate :  $\lim_{n \rightarrow \infty} \frac{n^2}{(n-7)^2 - 6}$

5. (a) If  $0 < x < 1$ , then show that  $\sum_{n=0}^{\infty} x^n$  converges to  $\frac{1}{1-x}$ .

(b) If  $x \geq 1$ , then show that  $\sum_{n=0}^{\infty} x^n$  diverges.

6. Prove that  $\sum_{n=1}^{\infty} \frac{1}{n}$  diverges.

7. Evaluate  $\int_0^1 x^7(1-x)^8 dx$ .

8. If  $u = \log(x^3 + y^3 + z^3 - 3xyz)$  show that  
 $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$ .

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If  $y = \sin^{-1} x$ , prove that  
 $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ .

10. Find an evolute of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

11. If  $I_n = \int_0^{\frac{\pi}{2}} x^n \cos x \, dx$ , then show that

$$I_n + n(n-1)I_{n-2} = \left[ \frac{\pi}{2} \right]^n.$$

12. If  $\{s_n\}_{n=1}^{\infty}$  is a cauchy sequence of real numbers, then show that  $\{s_n\}_{n=1}^{\infty}$  is bounded.

13. State and prove Ratio test.

14. Prove that  $\left\{ \left( 1 + \frac{1}{n} \right)^n \right\}_{n=1}^{\infty}$  is convergent.

15. Discuss the maxima and minima of the function  $x^3y^2(6-x-y)$ .

16. Change the order of integration  $\int_0^a \int_{x^2/a}^{2a-x} xy \, dx \, dy$  and evaluate it.

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