UG–329 BMS–12/BMC–12

B.Sc. DEGREE EXAMINATION –
DECEMBER, 2019.

First Year

Maths

TRIGONOMETRY, ANALYTICAL GEOMETRY AND VECTOR CALCULUS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that

$$\frac{\sin 7\theta}{\sin\theta} = 64\cos^6\theta - 80\cos^4\theta + 24\cos^2\theta - 1.$$

2. Find the image of the point (1, –2, 3) in the plane $2x - 3y + 2z + 3 = 0$.

3. Find the equation of the sphere passing through the four points $(2, 3, 1), (5, -1, 2), (4, 3, -1)$ and $(2, 5, 3)$.

4. Find the constants a, b, c so that the vector

$$\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$$

is irrotational.

5. Evaluate $\iiint_V \Delta \cdot \vec{F} dv$ if $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and if V is the volume of the region enclose by the cube $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$.

6. If $\sin(A + iB) = x + iy$, prove that

(a) $\dfrac{x^2}{\cosh^2 B} + \dfrac{y^2}{\sinh^2 B} = 1$

(b) $\dfrac{x^2}{\sin^2 A} - \dfrac{y^2}{\cos^2 A} = 1$

7. Find the symmetrical form of the equations of the lines $x + 5y - z - 7 = 0,\ 2x - 5y + 3z + 1 = 0$.

8. If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, evaluate $\int_c \vec{F} \cdot d\vec{r}$, where c is the curve on the xy plane $y = 2x^2$ from (0, 0) to (1, 2).

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Sum to infinity the series

$$\cos\alpha + \frac{1}{2}\cos(\alpha+\beta) + \frac{1.3}{2.4}\cos(\alpha+2\beta) + \ldots.$$

10. Prove that the lines

$$\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}; \frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$$

are co-planar. Find also their point of entersection and the plane through them.

11. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0$, $2x - y + 2z = 5$ for a great circle.

12. Find the directional derivative of $xyz - xy^2z^3$ at the point (1, 2, –1) in the direction of the vector $\vec{i} - \vec{j} - 3\vec{k}$.

13. Verify Green's theorem in the plane for $\int (xy - x^2)\,dx + x^2y\,dy$, where C is the boundary of the region bounded by $y = 0$, $x = 1$, $y = x$.

14. Prove that

$$\sin^3\theta\cos^5\theta = \frac{-1}{2^7}(\sin 8\theta + 2\sin 6\theta - 2\sin 4\theta - 6\sin 2\theta)$$

15. Find the perpendicular distance from (3, 9, –1) to the line $\frac{x+8}{-8} = \frac{y-31}{1} = \frac{z-13}{5}$.

16. Evaluate $\iint_s \vec{F}\cdot\hat{n}ds$, where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and s is the surface of the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$.

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