UG–331	**BMC–21/ BMS–21**

B.Sc. DEGREE EXAMINATION – JUNE, 2019.

Second Year

Mathematics with Computer Applications

GROUPS AND RINGS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Define (a) partial ordering relation, (b) poset and give examples.

2. Define centre of a group and normalizer of a group.

3. If H and K are subgroups of a group G, then show that $H \cap K$ is also a subgroup of G.

4. Let H be a subgroup of a group G. Show that the number of left cosets of H is the same as the number of right cosets of H.

5. Define a normal subgroup of a group. Show that every subgroup of an abelian group is a normal subgroup.

6. Let R be a ring with identity. Show that the set of all units in R is a group under multiplication.

7. Show that a finite commutative ring R without zero divisor is a field.

8. Show that the field of complex numbers is not an ordered field.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE of the following.

9. Define a bijection. Let $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x - 3$. Show that f is a bijection and compute f^{-1}. Also compute $f^{-1} \circ f$ and $f \circ f^{-1}$.

10. Show that a non-empty subset H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$.

11. State and prove Lagrange's theorem.

12. State and prove Cayley' s theorem.

13. Let R be a commutative ring with identity. Show that an ideal M of R is maximal if and only if R/M is a field.

14. (a) Show that any field is an integral domain.

    (b) Show that any finite integral domain is a field.

15. Show that any integral domain D can be embedded in a field F and every element of F can be expressed as a quotient of two elements of D.

16. Show that any Euclidean domain is a unique factorization domain.

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