UG–459	BMC–21

B.Sc. DEGREE EXAMINATION – DECEMBER, 2018.

Second Year

Mathematics with Computer Applications

GROUPS AND RINGS

Time : 3 hours | Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

1. Let ρ be an equivalence relation on a set S. Show that the set of all equivalence classes forms a partition on S.

2. Show that any permutation can be expressed as a product of disjoint cycles.

3. Show that a non-empty subset H of a group G is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$.

4. Let H be a subgroup of G. Show that the number of left cosets of H of H in G is the same as the number of right cosets of H in G.

5. Show that every group of prime order is cyclic.

6. Define a ring and give an example.

7. Show that any finite integral domain is a field.

8. If a be a non-zero element of a Euclidean domain R. Show that a is a unit if and only if $d(a) = d(1)$.

SECTION B — $(5 \times 10 = 50$ marks)

Answer any FIVE of the following.

9. Define :
   (a) Reflexive relation
   (b) Transitive relation
   (c) Partial ordering relation
   (d) Totally ordered set
   (e) Bijection.

10. Show that the union of two subgroups of a group G is a subgroup of G if and only if one is contained in another.

11. Define a cyclic group and show that a subgroup of a cyclic group is cyclic.

12. State and prove Cayley's theorem.

13. State and prove the fundamental theorem of group homomorphism.

14. Define a field. Show that a finite commutative ring R without zero-divisors is a field.

15. Let R be a commutative ring with identity. Show that an ideal M of R is maximal if and only if R/M is a field.

16. Show that any integral domain can be embedded in a field F and every element of F can be expressed as a quotient of two elements of D.