UG–339	BMS–35

B.Sc. DEGREE EXAMINATION — JUNE, 2019.

Third Year

Mathematics

GRAPH THEORY

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Define an isomorphism of graphs and give an example.

2. Show that every graph is an intersection graph.

3. Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

4. Prove that every tree has a centre consisting of either one point or two adjacent points.

5. If G is a graph in which the degree of each vertex is at least 2, then prove that G contains a cycle.

6. Prove that any subset of an independent set is independent.

7. Prove that a map G is 2-face colourable if and only if G is eulerian.

8. If G is a tree with n-points $n \geq 2$, then prove that $f(G, \lambda) = \lambda(\lambda - 1)^{n-1}$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Show that the sum of the degrees of the vertices of a graph is equal to twice the number of its edges.

10. If A is the adjacency matrix of G, then prove that the number of (v_i, v_j)-walks of length k in G is the $(i, j)^{\text{th}}$ entry of A^k.

11. Show that every non-trivial connected graphs has at least two points which are not cut points.

12. Let G be a (p, q) graph. Prove that the following statements are equivalent.

    (a) G is a tree

    (b) Every two points of G are joined by a unique path

    (c) G is connected and $p = q + 1$

    (d) G is acyclic and $p = q + 1$.

13. Prove that $C(G)$ is well defined.

14. If G is a graph with $p \geq 3$ vertices and $\delta > \frac{p}{2}$ then prove that G is Hamiltonian.

15. If G is a connected plane graph having V, E and F as the sets of vertices, edges and faces respectively, then show that $|V| - |E| + |F| = 2$.

16. Prove that every tournament has a spanning path.
