UG–335	BMS–32/BMC-32

B.Sc. DEGREE EXAMINATION —
DECEMBER, 2019.

Third Year

Mathematics

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Let V be a vector space over a field F. Prove that

   (a) $\alpha 0 = 0$ for all $\alpha \in F$.

   (b) $0v = 0$ for all $v \in V$.

   (c) $(-\alpha)v = \alpha(-v) = -(\alpha v)$ for all $\alpha \in F$ and $v \in V$.

   (d) $\alpha v = 0 \Rightarrow \alpha = 0$ or $v = 0$.

2. Let A and B be subspace of a vector space V. Prove that $A \cap B = \{0\}$ if and only if every vector $v \in A + B$ can be uniquely expressed in the form $v = a + b$ where $a \in A$ and $b \in B$.

3. Find the linear transformation $T : V_3(R) \to V_3(R)$ determined by the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ with respect to the standard basis $\{e_1, e_2, e_3\}$.

4. Let $T : V \to W$ be a linear transformation. Prove that $\dim V = rank\ T + nullity\ T$.

5. Let V be a vector space of polynomials with inner product given by $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$. If $f(t) = t + 2$, and $g(t) = t^2 - 2t - 3$, find $\langle f, g \rangle$.

6. Let V be a finite dimensional inner product space and let W be a subspace of V. Prove that $\left(W^{\perp}\right)^{\perp} = W$.

7. Let f be a symmetric bilinear form defined on V and let q be the associated quadratic form. Prove that $f(u, v) = \frac{1}{4}[q(u+v) - q(u-v)]$.

8. Let G be a group and let L be the set of all subgroups of G. In L we define $A \leq B$ if and only if $A \subseteq B$. Prove that L is a lattice.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let V and W be vector spaces over a field F. Let $L(V,W)$ represent the set of all linear transformation from V to W. Prove that $L(V,W)$ itself is a vector space over addition and scalar multiplication defined by $(f+g)(v) = f(v) + g(v)$ and $(\alpha f)(v) = \alpha f(v)$.

10. Let V be a vector space over a field F. Let $S, T \subseteq V$. Prove that

(a) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

(b) $L(SUT) = L(S) + L(T)$.

11. If W is a subspace of a finite dimensional vector space V, show that $\dim \frac{V}{W} = \dim V - \dim W$.

12. Let W_1 and W_2 be subspaces of a finite dimensional inner product space. Prove that

(a) $(W_1 + W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$.

(b) $(W_1 \cap W_2)^{\perp} = W_1^{\perp} + W_2^{\perp}$.

13. Apply Gram – Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$ where $v_1 = \{1, -1, 0\}$, $v_2 = \{2, -1, -2\}$ and $v_3 = \{1, -1, -2\}$.

14. Let V be a vector space over a field F. Prove that $L(V,V,F)$ is a vector space over F under addition and scalar multiplication defined by

    (a) $(f+g)(u,v) = f(u,v) + g(u,v)$ and

    (b) $(\alpha f)(u,v) = \alpha f(u,v)$, where $L(V,V,F)$ is the set of all bilinear form on V.

15. Reduce the quadratic form $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$ to the diagonal form.

16. (a) Let B be a Boolean algebra. Show that $(a \vee b)' = a' \wedge b'$, $(a \wedge b)' = a' \vee b'$ and $(a')' = a$.

    (b) In a Boolean algebra if $a \vee x = b \vee x$ and $a \vee x = b \vee x$ then show that $a = b$.

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