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| **UG–334** | **BMS–31/ BMC–31** |

B.Sc. DEGREE EXAMINATION — JUNE, 2019.

Third Year

Mathematics with Computer Applications

# REAL AND COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 75

## SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that the interval [0, 1] is uncountable.

2. Define an open set and prove that in any metric space every open ball is an open set.

3. Define continuity of a function at a point. Examine whether the function $f(x)=\begin{cases}\frac{|x|}{x}, & x \neq 0 \\ 0, & x=0\end{cases}$ is continuous or not?

4. Find an open covering of $E=(0,1)$ which does not contain a finite sub-covering.

5. Prove that every continuous function is Riemann-integrable.

6. State and prove Rolle's theorem.

7. Find the bilinear transformation which map points $z = 0, -i, -1$ into $w = i, 1, 0$.

8. State and prove Cauchy residue theorem.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that the set of all real numbers with usual metric is of second category.

10. Prove that the continuous image of a compact set is compact.

11. Prove that a subset of $R$ is connected if and only if it is an interval.

12. State and prove chain rule for differentiable functions.

13. State and prove first fundamental theorem of integral calculus.

14. Show that the inversion transformation $w = \frac{1}{z}$ transforms circles in to circles or straight lines.

15. State and prove Cauchy Riemann equations.

16. Using Cauchy residue theorem/evaluate $\int_0^{2\pi} \frac{1}{a + b\cos\theta} d\theta, a > |b| > 0.$